MAP-BASED MUON COOLING CHANNEL SIMULATIONS WITH COSY INFINITY

KYOKO MAKINO
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA E-mail: makino@uiuc.edu

MARTIN BERZ
Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA E-mail: berz@msu.edu

The Differential Algebraic (DA) technique allows to obtain high order transfer maps of particle optical systems in an efficient way. The DA based normal form algorithm, developed and implemented in the code COSY Infinity, provides a mechanism to diagnose and optimize the nonlinear behavior of repetitive systems. The proposed cooling channels for muon accelerators are long chains of cells comprised of accelerating cavities, energy absorbers and strong solenoids, and the DA based normal form algorithm is applicable to optimize them. This note will provide an overview of the method and report the first simulation of a muon cooling channel with COSY Infinity.

1 Introduction

Efficient muon beam cooling is critical to the eventual success of both neutrino factories and muon colliders, and has to be sufficiently demonstrated before further development.

The transfer map method has several advantages compared to ray tracing methods, and the Differential Algebraic (DA) technique brought an efficient and elegant way to compute high order Taylor transfer maps [1,2,3]. First of all, the transfer map allows the direct computation of relevant characteristics of the system. Secondly, it is not necessary to integrate through a repeating part of the system again and again; instead, one just computes the maps for the repeating pieces, then concatenates them in the appropriate order. Finally, even for mere tracking there is some advantage in that the once-computed transfer map is merely applied to the particle coordinates repeatedly.

COSY Infinity [4] has the DA technique built in at its "COSY" language level, and various DA based algorithms have been implemented [4,5]. The DA normal form method [6,7,3] is one of them, and it provides an excellent mechanism to analyze repetitive systems. The next section presents an overview of the DA normal form method. Combined with the optimization capability in COSY Infinity, the design task of repetitive systems can be performed in a systematic and efficient manner.

Currently contemplated beamlines for muon cooling are long channels of cells, each cell consisting of solenoids, accelerating cavities and energy absorbers. The whole channel can be viewed as a repetitive system, thus the

DA normal form method can assist for the system optimization. In the subsequent sections, we will discuss the first simulation result with COSY Infinity for such a muon cooling channel with a simple model, and further establish a strategy for the optimization of the proposed muon cooling channel [8].

2 Differential Algebraic Normal Form Methods

The DA normal form algorithm consists of a series of coordinate transformations. The idea is to perform a nonlinear change of variables such that the motion in the new variables becomes highly regular. Indeed, in the simplest non-resonant case, without damping it just follows an approximately circular motion, while any damping present leads to a spiral-like structure. Since a mere change of coordinates does not affect the general topological properties, this approach usually allows for a much more detailed analysis of the motion. The details of the theory can be found in [6,7,3].

The following tracking pictures show the benefit of the normal form method straightforwardly. Figure 1 shows tracking pictures for a non-damped system, and Figure 2 for a damped system. The tracking picture displayed in normal form coordinates for a non-damped system (right in Figure 1) shows almost perfect regularity, indicating the system is very stable. On the other hand, it is hard to diagnose the stability of the system in conventional coordinates (left in Figure 1). For the damped system, the tracking picture in normal form coordinates (right in Figure 2) shows weak damping and stability of the system. In conventional coordinates, the phase space positions of particles are influenced strongly by the nonlinearities, so one cannot diagnose the stability of the system. A muon cooling channel is obviously a damped system, so the DA normal form method is expected to simplify the analysis of the system.

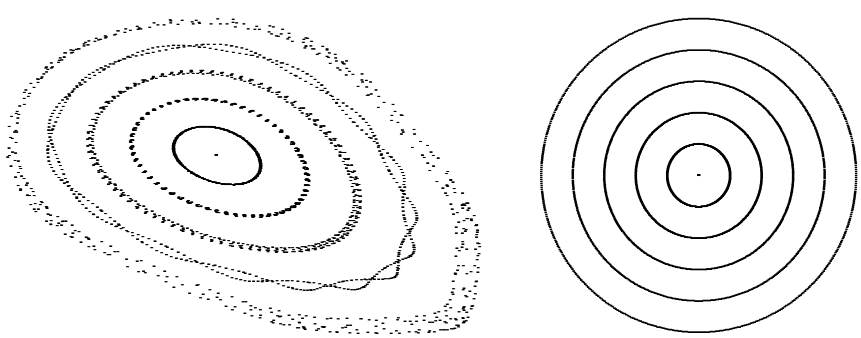

Figure 1. Tracking pictures of a non-damped system displayed in conventional coordinates (left), and in normal form coordinates (right).

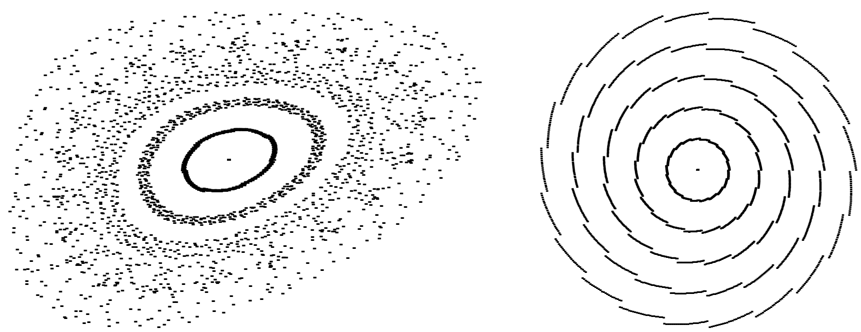

Figure 2. Tracking pictures of a damped system displayed in conventional coordinates (left), and in normal form coordinates (right).

3 Example

A simple system was set up to test the simulation with COSY Infinity for a muon cooling channel. As seen from the system parameters listed in Table 1, the cell (A) consists of a solenoid, an absorber and an energy compensating cavity, and the cell (B) consists of a solenoid only. Both cells are set to have the same total length. The transfer maps were computed once for both cells (A) and (B) for 250 MeV muon beams.

The test particles, ten particles starting at the x positions 3cm, 6cm, ..., 30cm, were sent through the cells and were tracked for the first twenty cells, and for the twenty cells after the first one hundred cells, i.e. the 101th through 120th cells, to see the system behavior after a suitable time. The tracking pictures for the first twenty cells, shown in the left pictures in Figure 3 for the system (A) and in Figure 4 for the system (B), do not exhibit a clear difference. At the later time, i.e. after the first one hundred cells, the particles

Table 1. System parameters of simple test cells.

(A) Cooling Cell		(B) Non-Cooling Cell	
Drift	0.175m	Drift	0.175m
Solenoid	0.167m Radius 0.33m, Field 2T	Solenoid	0.167m Radius 0.33m, Field 2T
Drift	0.6m	Drift	1.0m
Absorber	0.2m Energy Loss 30MeV/m		
Cavity	0.2m Energy Gain 30MeV/m		

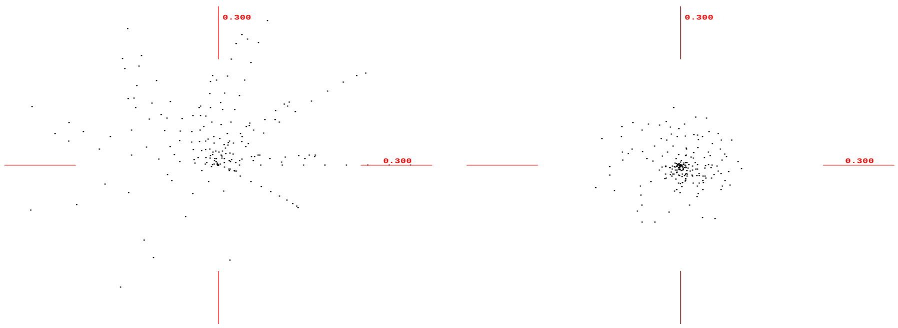

Figure 3. Tracking pictures of a simple muon cooling system (A) for the first 20 cells, and for the 20 cells after the first 100 cells. The pictures show the x-y motion for 10 particles starting at x 3cm, 6cm, ..., 30cm.

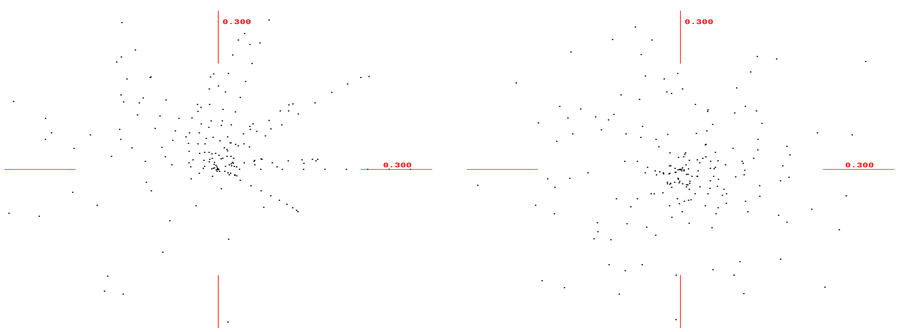

Figure 4. Tracking pictures of a solenoid system (B) for the first 20 cells, and for the 20 cells after the first 100 cells. The pictures show the x-y motion for 10 particles starting at x 3cm, 6cm, ..., 30cm.

were tracked for the additional twenty cells as shown in the right pictures in Figures 3 and 4. In the non-cooling system (B), the particle positions in the x-y space are scattered in the whole area of the picture. On the other hand, in the system (A), the particles are well damped showing the effect of the beam cooling.

4 Outlook

As discussed earlier, all the necessary Differential Algebra based computational tools for the efficient simulation of a muon cooling channel optimization are available in the code COSY Infinity. The principle of a muon cooling channel was simulated with COSY Infinity using a simple cooling cell, showing the visible cooling effect in the tracking pictures.

We are currently in the process of developing more general cell elements representing all details of the actual designs that appear in muon cooling channels such as those in the Feasibility II Study [8]. For example, in an actual muon cooling channel, accelerating cavities and energy absorbers are placed between and even inside solenoids. It is necessary to treat the solenoids, and the cavities or the absorbers efficiently but in their proper superposition. Since the apertures of solenoids are large compared to the length in the beam axis direction, the fields created by separate individual solenoids are not independent from each other anymore. Rather it is necessary to model a continuously superimposed solenoidal field, and one has to be able to accommodate the insertion of the cavities and the absorbers. The windows of currently proposed absorber chambers facing to the beam axis have a spherical shape, which also has to be accounted for.

Once those detailed elements are represented in the code, we can use the advantages of the transfer map approach combined with the normal form method to identify which are critical parameters to be optimized.

Acknowledgments

The work was supported in part by the Illinois Consortium for Accelerator Research under contract FY-2001-IBHE-HECA and in part by the US Department of Energy under contract DE-FG02-95ER40931 and an Alfred P. Sloan Fellowship.

References

1. M. Berz in *Physics of Particle Accelerators*, ed. M. Month (American Institute of Physics, 1989).
2. M. Berz in *Automatic Differentiation of Algorithms: Theory, Implementation and Application*, ed. A. Griewank and G.F. Corliss (SIAM, Philadelphia, 1991).
3. M. Berz, *Modern Map Methods in Particle Beam Physics* (Academic Press, San Diego, 1999).
4. M. Berz, *COSY INFINITY Version 8 Reference Manual, MSUCL-1088* (NSCL, Michigan State University, East Lansing, MI 48824, 1997). See also http://cosy.nscl.msu.edu.
5. K. Makino and M. Berz in *Fourth Computational Accelerator Physics Conference*, (AIP Conference Proceedings, 1996).
6. M. Berz in *Proc. Nonlinear Effects in Accelerators*, ed. M. Berz, S. Martin and K. Ziegler (IOP Publishing, London, 1992).
7. M. Berz and K. Makino, *International Journal of Computer Research*, in print (2001).
8. Muon Collaboration, *Feasibility Study II* (2001).